Exercise 4

Repeat Exercise 3 using the function in Figure 10.

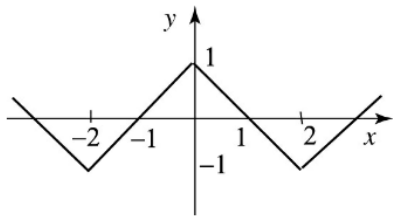

Figure 10 for Exercise 4.

Solution

Part (a)

Notice that the function repeats itself every 4 units, so the period is 4.

$$f(x) = \begin{cases} 1+x & \text{if } -2 \leq x \leq 0 \\ 1-x & \text{if } 0 \leq x \leq 2 \\ f(x+4) & \text{otherwise} \end{cases}$$

Part (b)

The function is continuous for all x, so $f(x-) = f(x+)$ everywhere, which means the function is piecewise continuous on $-2 \leq x \leq 2$.

Part (c)

The derivative of f is undefined wherever f has kinks in the graph.

$$f'(x) = \begin{cases} \frac{d}{dx}(1+x) & \text{if } -2 < x < 0 \\ \frac{d}{dx}(1-x) & \text{if } 0 < x < 2 \\ f'(x+4) & \text{otherwise} \end{cases}$$

$$= \begin{cases} 1 & \text{if } -2 < x < 0 \\ -1 & \text{if } 0 < x < 2 \\ f'(x+4) & \text{otherwise} \end{cases}$$

The function's derivative is continuous everywhere except at every even integer.

$$\{x \mid x \neq 2k, \quad k = 0, \pm 1, \pm 2, \ldots\}$$

Compute the limits of the derivative at the discontinuities.

$$\lim_{x \to (2k)^-} f'(x) = \begin{cases} -1 & \text{if } k \text{ is odd} \\ 1 & \text{if } k \text{ is even} \end{cases}$$

$$\lim_{x \to (2k)^+} f'(x) = \begin{cases} 1 & \text{if } k \text{ is odd} \\ -1 & \text{if } k \text{ is even} \end{cases}$$

The function's derivative is piecewise continuous on $-2 \le x \le 2$ because $f'(-2+)$ and $f'(2-)$ exist, and there are only a finite number of discontinuities in $-2 < x < 2$ that each have existing one-sided limits. Therefore, f is piecewise smooth on $-2 \le x \le 2$.